DOA Estimation with Triply Primed Arrays Based on Fourth-Order Statistics

Kai-Chieh Hsu and Jean-Fu Kiang*

Department of Electrical Engineering

Nation Taiwan University

Outline

- Introduction
- Proposed methods
- Simulations and discussions
- Conclusion

DOA estimation algorithms

- Conventional algorithms
  - Multiple Signal Classification (MUSIC) [1]
  - Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [2]
- Increase degrees of freedom (DOFs)
  - Khatri-Rao subspace (KR) [3]
  - Co-prime array (CPA) [4], [5]

[1] R. Schmidt, “Multiple emitter location and signal parameter estimation,” *IEEE Trans. Antennas Propagat.*, vol. 34, no.3, pp. 276-280, 1986.
[2] R. Roy and T. Kailath, “ESPRIT-estimation of signal parameters via rotational invariance techniques,” *IEEE Trans. Acous. Speech Signal Process.*, vol. 37, no.7, pp. 984-995, 1989.
[3] W. K. Mao, T. H. Hsieh and C. Y. Chi, “DOA estimation of quasi-stationary signals with less sensors than sources and unknown spatial noise covariance: A Khatri-Rao subspace approach,” *IEEE Trans. Signal Process.*, vol. 58, no. 4, pp. 2168-2180, 2010.
[4] P. Pal and P. P. Vaidyanathan, “Coprime sampling and the MUSIC algorithm,” *Proc. IEEE DSP/SPE* Workshop, pp. 289-294, 2011.
[5] S. Qin, Y. D. Zhang and M. G. Amin, “Generalized coprime array configurations for direction-of-arrival estimation,” *IEEE Trans. Signal Process.*, vol. 63, no. 6, pp. 1377-1390, 2015.

Co-prime Array (CPA)

- Array configuration
  - Subarray 1 is composed of $2N_1$ sensors at spacing $N_2 d$.
  - Subarray 2 is composed of N_2 sensors at spacing $N_1 d$.
  - Total $2N_1 + N_2 - 1$ sensors in the CPA.

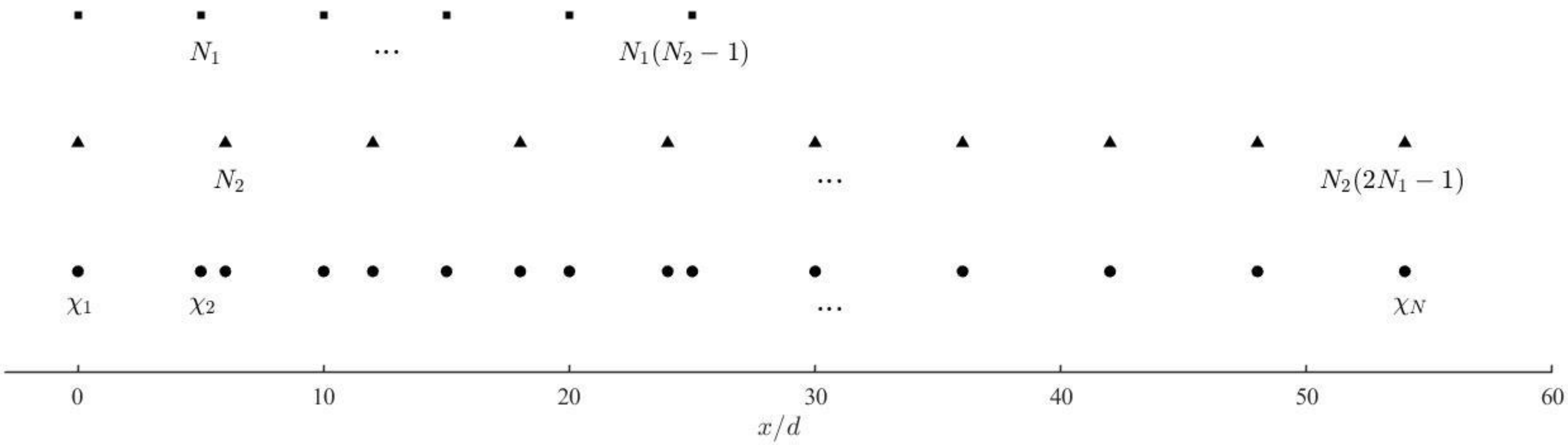
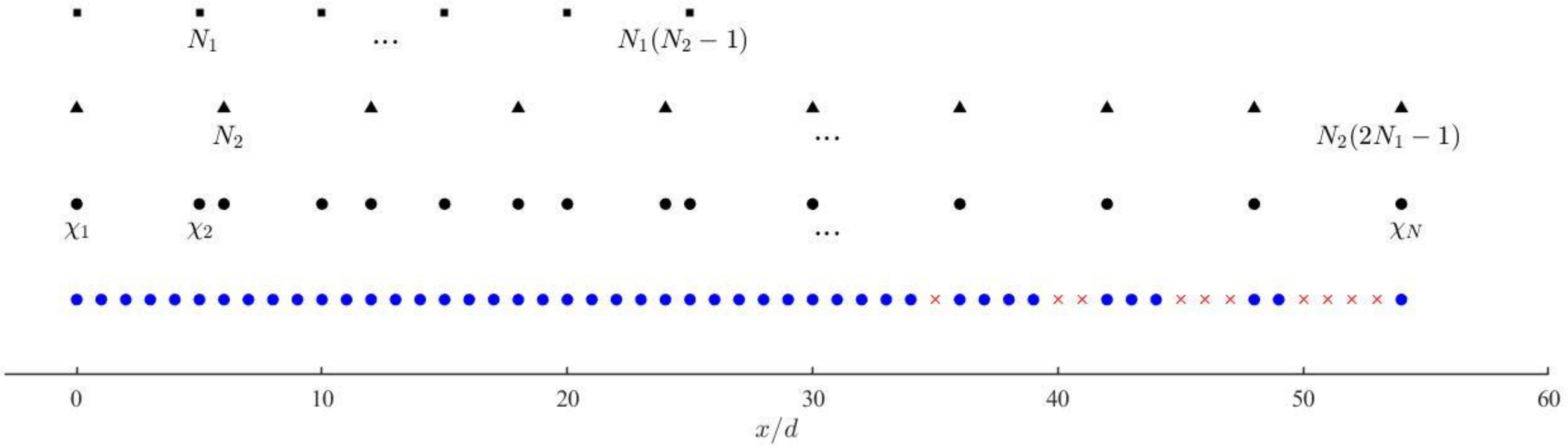


Fig. 1. Configuration of CPA

DOF of Co-prime Array

- DOFs are increased to
  - $3N_1N_2 + N_1 - N_2$ unique virtual sensors
  - $N_1N_2 + N_1 - 1$ consecutive virtual sensors



Proposed triply primed array (TPA)

- We propose a new array configuration, triply primed array (TPA)
- The DOF is further extended to $O(N_1 N_2 N_3)$.
- We propose a dimension-reduced method to significantly decrease the computational complexity.

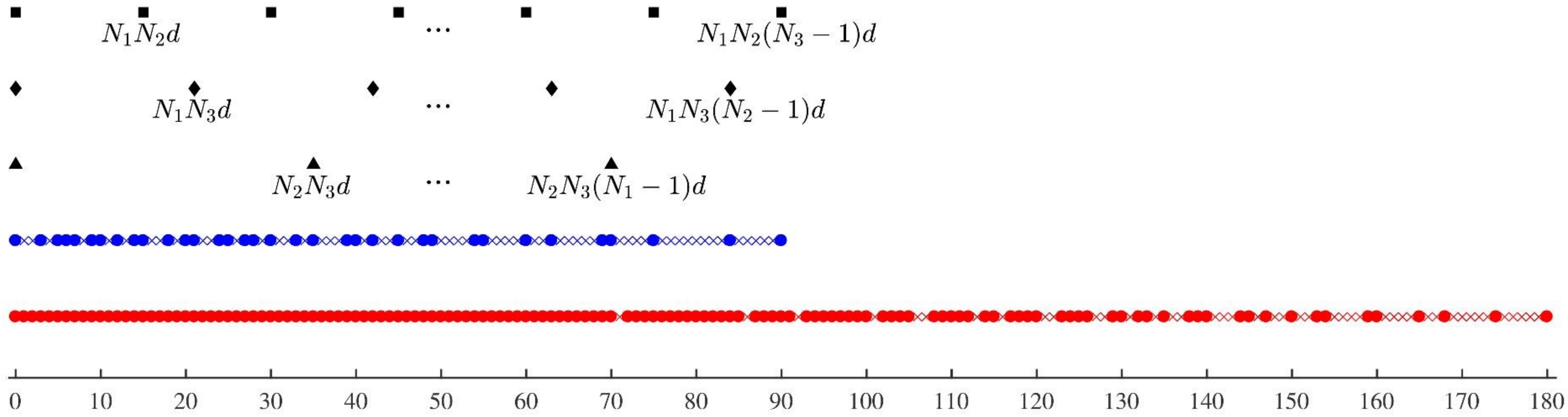


Fig. 2. configuration of TPA

Configuration of TPA

- A TPA is composed of three subarrays, with N_1, N_2 and N_3 sensors, at spacing N_2N_3d, N_1N_3d and N_1N_2d.
- Total number of physical sensors: $N = N_1 + N_2 + N_3 - 2$.

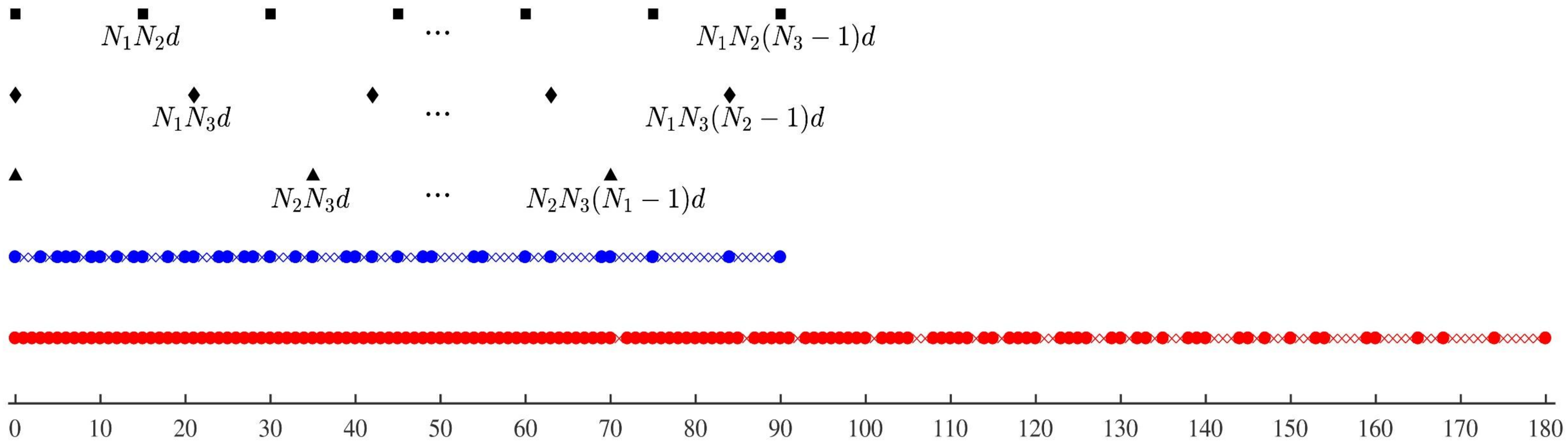


Fig. 2. configuration of TPA

Properties of TPA

- The number of unique lags:

$$\tau_u = 2N_1N_2N_3 + N_1N_2 + N_2N_3 + N_3N_1 - 8$$

- The number of positive consecutive lags:

$$\tau_c = N_1N_2 + N_2N_3 + N_3N_1 - 1$$

- By using fourth-order difference, higher DOFs are achieved in unique lags and consecutive lags.

Signal Model

- Received data: $\bar{\bar{X}} \in \mathbb{C}^{N \times LQ}$, where Q is the number of observed time frames and L is the length of each time frame.
- Each column of $\bar{\bar{X}}$ is composed of received signals from three subarrays $\bar{x}[\ell] = \bar{\bar{A}} \cdot \bar{s}[\ell] + \bar{n}[\ell]$,

$$\bar{x}[\ell] = \begin{bmatrix} \bar{x}_1[\ell] \\ \bar{x}_2[\ell] \\ \bar{x}_3[\ell] \end{bmatrix}, \quad \bar{\bar{A}} = \begin{bmatrix} \bar{\bar{A}}_1 \\ \bar{\bar{A}}_2 \\ \bar{\bar{A}}_3 \end{bmatrix}, \quad \bar{n}[\ell] = \begin{bmatrix} \bar{n}_1[\ell] \\ \bar{n}_2[\ell] \\ \bar{n}_3[\ell] \end{bmatrix}$$

where $\bar{\bar{A}}_1$, $\bar{\bar{A}}_2$ and $\bar{\bar{A}}_3$ are the steering matrices of the three subarrays, $\bar{n}_1[\ell]$, $\bar{n}_2[\ell]$ and $\bar{n}_3[\ell]$ are i.i.d. noise vectors of the three subarrays.

Covariance matrix of TPA Signals

- Construct covariance matrix as $$\tilde{\bar{\bar{R}}}_{xxq} = \frac{1}{L} \sum_{\ell=(q-1)L}^{qL-1} \bar{x}^q[\ell]\bar{x}^{q\dagger}[\ell]$$

  which is composed of submatrices

$$\bar{\bar{R}}_{\alpha\beta q} = \mathrm{E}\{\bar{x}^q_\alpha[\ell]\bar{x}^{q\dagger}_\beta[\ell]\} = \bar{\bar{A}}_\alpha \cdot \bar{\bar{D}}_{xxq} \cdot \bar{\bar{A}}^\dagger_\beta + \sigma^2_\beta \bar{\bar{W}}_{\alpha\beta}$$

$$R_{12q}[u,v] = \begin{cases} \sum_{m=1}^{M} P_{qm} e^{jkN_3(uN_2 - vN_1)d\sin\theta_m} + \sigma_n^2, \\ \qquad u = v = 0 \\ \sum_{m=1}^{M} P_{qm} e^{jkN_3(uN_2 - vN_1)d\sin\theta_m}, \\ \qquad u \neq 0 \text{ or } v \neq 0 \end{cases}$$

Each entry corresponds to a sensor location in the virtual TPA

Second-order manifold signals

- Second-order manifold signals:

$$\bar{y}_q = \text{vec}\left\{ \bar{\bar{R}}_{xxq} \right\} \qquad \bar{y}_q'' = \bar{y}_q - \bar{y}_q' \qquad \mathrm{E}\left\{ \bar{y}_q \right\}$$

- The entries in a submatrix corresponding to the same lags are averaged.

Fourth-order manifold signals

- Construct fourth-order manifold signals as

$$\tilde{\bar{\bar{R}}}_{yy} = \frac{1}{Q}\sum_{q=1}^{Q} \bar{y}_q''\bar{y}_q''^{\dagger} \qquad \bar{z} = \text{vec}\left\{\bar{\bar{R}}_{yy}\right\}$$

- MUSIC or ESPRIT cannot be directly applied to $\bar{z}$
- Spatial-smoothing MUSIC (SSM) and compressed sensing (CS) methods are applied

Dimension-reduced method

- Before applying SSM or CS to solve for the DOA, a dimension-reduced method is used to
- The number of overlapped lags in the second-order covariance matrix is small, but that of the fourth-order covariance matrix is large.
- The overlapped lags of each entry in $\tilde{\bar{\bar{R}}}_{yy}$ is recorded in a dictionary.
- By taking the average of entries with the same lag, a dimension-reduced fourth-order manifold signal is formed as $\bar{z}_r$.

Spatial-smoothing MUSIC [4]

- Conventional SSM requires the received signal vector be derived from consecutive lags, which is extracted from $\bar{z}_r$ and represented as

$$\bar{z}_c = [z_c[-\tau_c], z_c[-\tau_c+1], \cdots, z_c[\tau_c-1], z_c[\tau_c]]^t$$

- An SSM matrix is constructed as

$$\bar{\bar{R}}_{zz,ss} = \frac{1}{\tau_c+1}\sum_{\xi=0}^{\tau_c} \bar{z}_{c\xi}\bar{z}_{c\xi}^{\dagger}$$

where $\bar{z}_{c\xi} = [z_c[\xi-\tau_c], z_c[\xi-\tau_c+1], \cdots, z_c[\xi-1], z_c[\xi]]^t, 0 \leq \xi \leq \tau_c$

- The MUSIC is then applied to the SSM matrix to estimate DOA.

Compressed sensing (CS) [5]

- Although the SSM takes low computational load, it does not make use of the receiving data from nonconsecutive lags and its resolution is limited.
- The CS approach can be applied to cope with the above shortcomings by exploiting the sparsity properties of source signals in the angular domain.
- We formulate an ℓ_1-norm optimization problem:

$$\bar{\psi}_g^{\circ} = \arg\min_{\bar{\psi}_g} \|\bar{\psi}_g\|_1 \quad \text{s.t.} \quad \|\bar{z}_r - \bar{\bar{\bar{C}}}'_{g,\Phi_C} \cdot \bar{\psi}_g\|_2 < \epsilon$$

where $\bar{\bar{\bar{C}}}'_{g,\Phi_C}$ is the steering matrix of the fourth-order virtual array at a specified resolution.

Simulation Setting

- The length of each time frame is randomly peak from U[300,700]
- The DOAs are at uniform spacing between $\left[-60^{\circ}, 60^{\circ}\right]$
- In each scenario, 100 Monte Carlo realizations are simulated.

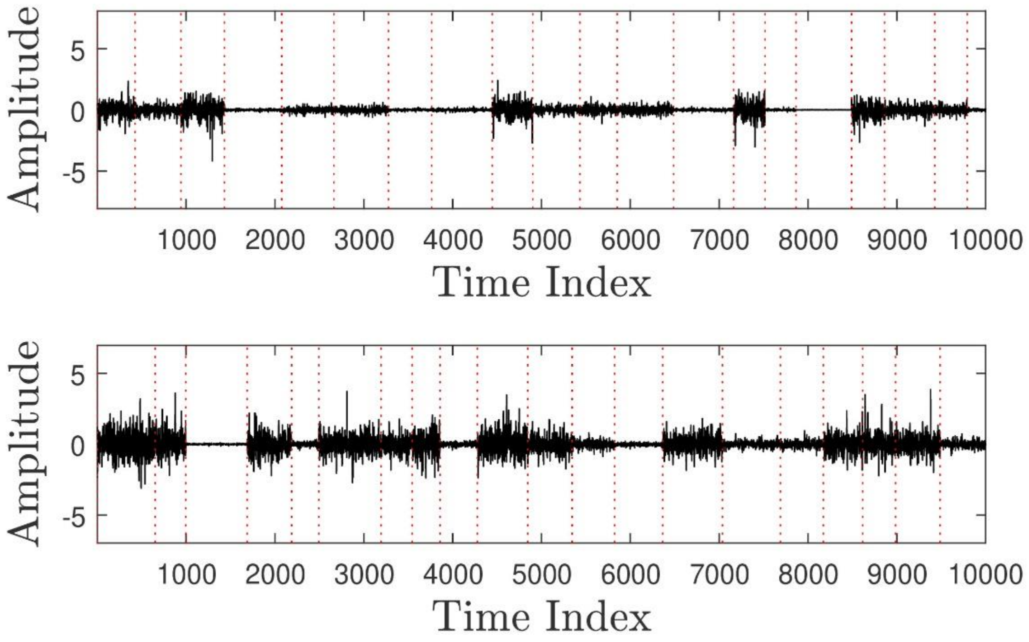


Fig. 3. Source Signals

Number of detected sources

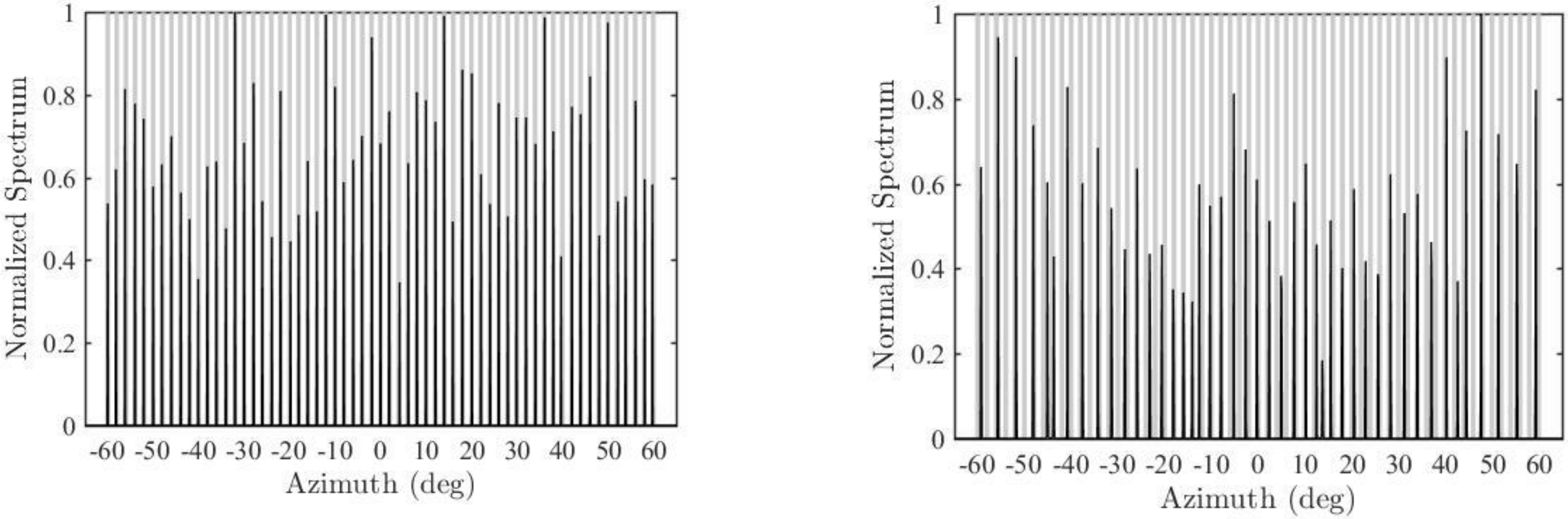


Fig. 4: Normalized spectrum of (left) TPA (3, 4, 5) and (right) CPA (3, 5), 61 sources, SNR = 5 dB, L = 500, Q = 1, 000.

Gray lines: actual DOAs, black lines: estimated DOAs.

TPA detect all 61 sources, CPA misses some sources and falsely identify some sources.

RMSE versus SNR

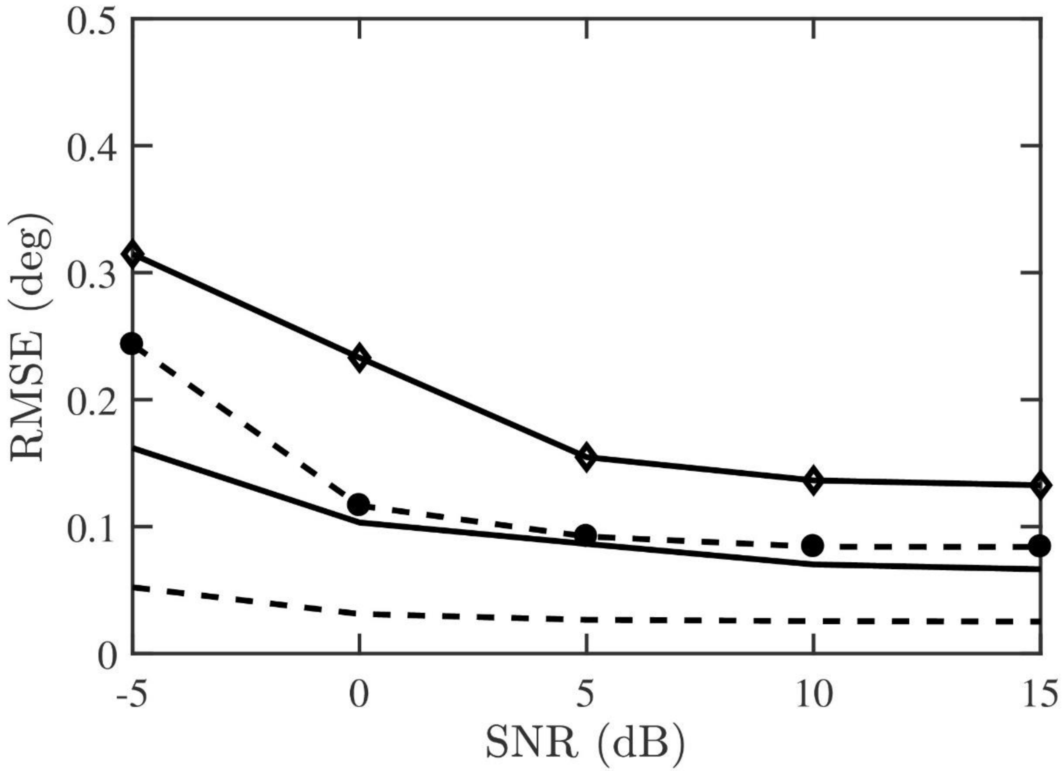


Fig. 5: RMSE of DOA estimation versus SNR, 36 sources, L = 500, Q = 1, 000.
———: TPA (3, 4, 5), CS; – • –: TPA(3, 4, 5), SSM; – ⋄ –: CPA(3, 5), CS; – – –: TPA(3, 5, 7), CS.

- TPA with either SS-MUSIC or CS approach predicts more accurate DOAs under all SNRs, especially when SNR < 0 dB.
- The CS approach predicts more accurate DOAs than SS-MUSIC because the former makes use of all the unique lags, but the latter can use only consecutive lags.

RMSE versus frame length

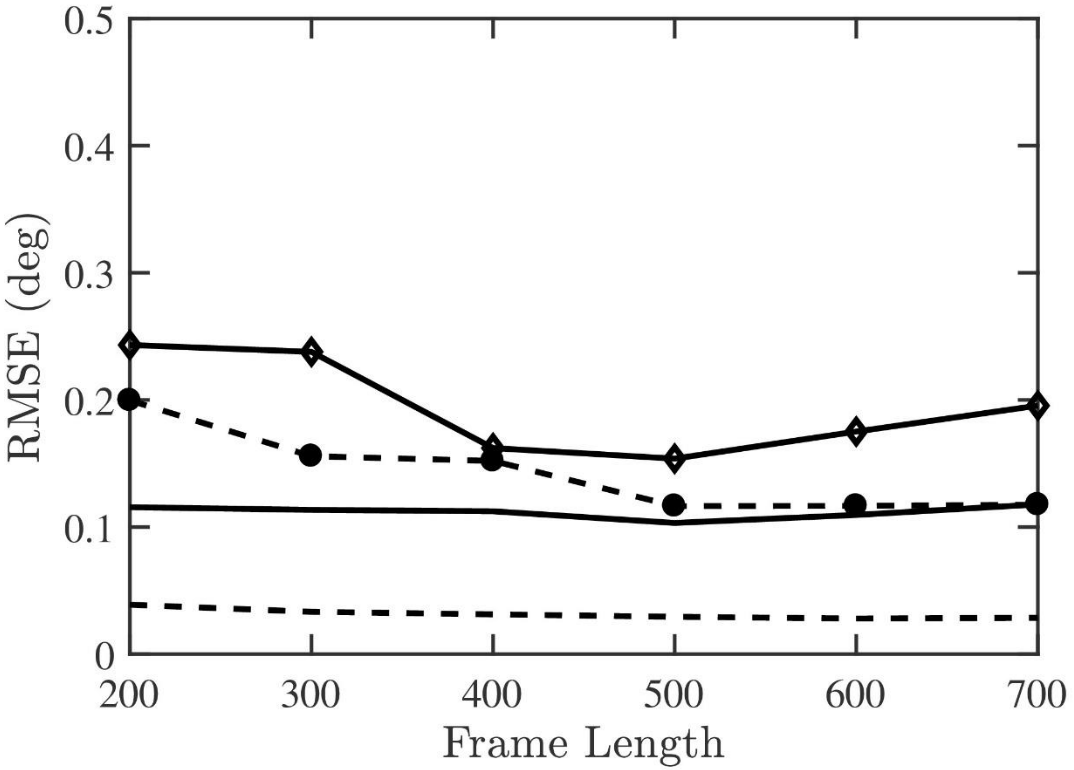


Fig. 6: RMSE of DOA estimation versus length of time frame, 36 sources, Q = 1, 000, SNR = 0dB.
—— : TPA (3, 4, 5), CS; – • –: TPA(3, 4, 5), SSM;
– ◊ – : CPA(3, 5), CS; – – –: TPA(3, 5, 7), CS.

- TPA with CS approach is hardly affected by the change of L due to higher DOF.
- The RMSE of CPA with CS approach and TPA with SS-MUSIC increases when L is different from 500, possibly because the estimation of power in each time frame becomes less accurate when L is different from 500.

RMSE versus number of frames

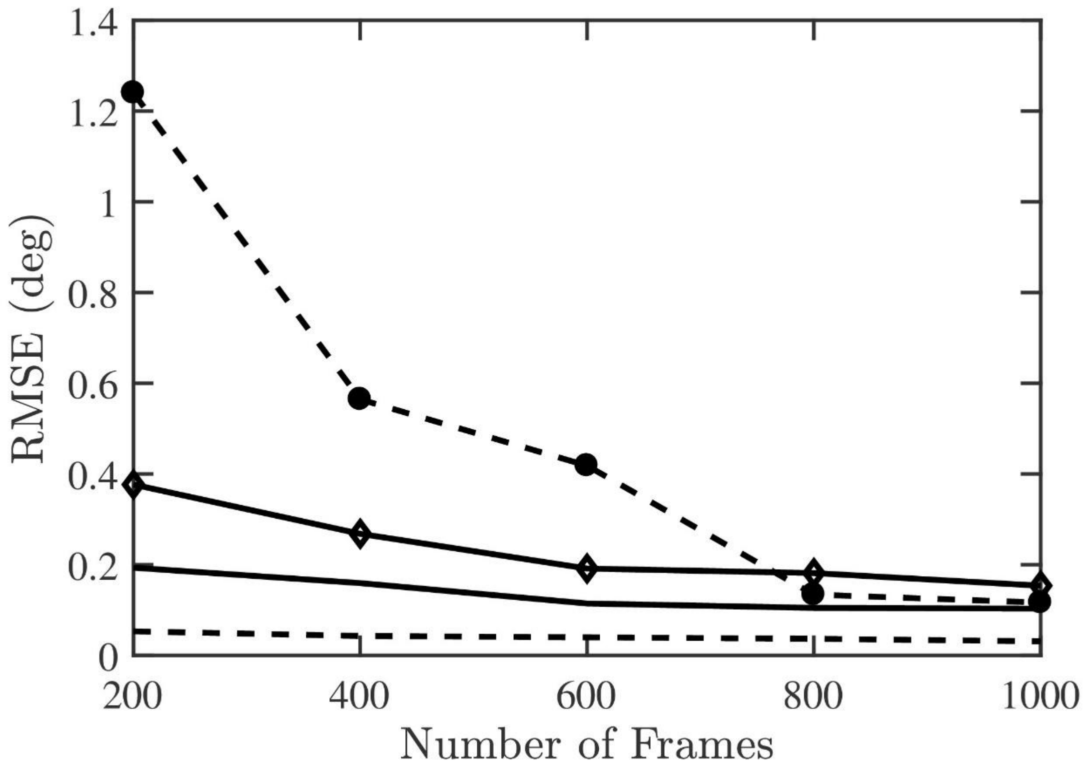


Fig. 7: RMSE of DOA estimations versus number of time frames, 36 sources, L = 500, SNR = 0dB.
——: TPA (3, 4, 5), CS; − • −: TPA(3, 4, 5), SSM;
− ⋄ −: CPA(3, 5), CS; − − −: TPA(3, 5, 7), CS.

- TPA with CS approach gives more accurate estimation than the other two, and the accuracy degrades monotonically when the number of frames decreases.
- SS-MUSIC is a subspace-based algorithm, which is more sensitive to the accuracy of covariance matrix. The covariance matrix can be estimated more accurately as the number of time frames increases.

Conclusion

- A TPA configuration is proposed to extend the DOFs in terms of the numbers of unique lags and consecutive lags.
- A dimension-reduced algorithm is proposed to speed up the algorithm.
- Simulation results show that the TPA can detect more sources than conventional CPA, and the RMSE is also lower.